BUBNOV-GALERKIN METHOD FOR SOLVING DIFFERENTIAL EQUATIONS

Pirnazarov Gulomjon Farhadovich, Odilova Dildora Shuhrat kizi
Tashkent State Transport University

Annotation: The bend is called flat or straight if the plane of action of the load passes through the main central axis of inertia of the section. As a result of the construction, a plot of the distribution of the bending moment for each section of the beam was obtained, from which the greatest value is visible. Changes in normal and tangential stresses along the length of the beam depend on the values of transverse forces and bending moments. This study allows you to gain skills that will be used in project practice.

Keywords: beams, axis, plane, curved axis, rod, beam, structure, cross sections.

The Bubnov-Galerkin method is one of the methods for solving differential equations, which was developed in the early 20th century by Russian mathematicians Sergei Bubnov and Boris Galerkin. This method is one of the most effective and widely used methods for solving differential equations.

The Bubnov-Galerkin method is based on the representation of the solution of a differential equation in the form of a linear combination of functions that are selected in such a way as to satisfy the initial and boundary conditions. These functions are called basis functions.

To apply the Bubnov-Galerkin method, it is necessary to select the basis functions that will be used to represent the solution of the differential equation. Usually, such functions are chosen as basis functions that satisfy certain conditions, for example, orthogonality or compactness.

Then it is necessary to substitute the representation of the solution into the original differential equation and obtain a system of equations for the coefficients of the linear combination. This system of equations is solved numerically, and the coefficients obtained are used to find a solution to the differential equation.

The Bubnov-Galerkin method is a numerical method for solving differential equations, which is based on the representation of the solution as a linear combination of a certain basis of functions. In this case, the basis can be functions describing the shape of the plate, for example, the functions of sines and cosines

Steps of the Bubnov-Galerkin method for calculating a pivotally fixed rectangular plate:

1. Set the basis of the functions that we will use to approximate the solution. In this case, you can use the sine and cosine functions.
2. Write down the equation describing the behavior of the plate. In this case, it is the equation of vibrations of an elastic plate.
3. Substitute the basis functions into the equation and obtain a system of equations for the coefficients of decomposition of the solution by the basis.
4. Solve a system of equations for coefficients.

5. Substitute the coefficients found in the expression for the solution and get a numerical solution to the problem.

The Bubnov-Galerkin method makes it possible to obtain an approximate solution of a differential equation with high accuracy using a finite number of basis functions. However, to achieve high accuracy, it is required to use a large number of basic functions, which can lead to computational difficulties.

The Bubnov-Galerkin method is based on a theorem from the theory of general Fourier series. Let {Ui} be a complete system of functions (with nonzero norm) orthogonal on the segment [a,b]. If a continuous function f(x) is also orthogonal on the segment [a,b] to all functions Ui(x), i.e.

$$\int_a^b f(x)\ \varphi_i\ (x)\ \ dx = 0$$

then the function f(x) is identically zero for a<=x<=b.

According to the Galerkin method , the solution is also sought in the form:

$$Y= \varphi_0\ (x) + \sum_{i=1}^{n} a_i\ \varphi_i\ (x)\ (1)$$

Where satisfies $\varphi_o\ (x)$ inhomogeneous (non-zero) boundary conditions:

$$\varphi_o\ (a) = A,\ \varphi_o\ (b) = B\ (2)$$

And the function (i-1,2,....,n) satisfies (zero) boundary conditions.

$$\varphi_i\ (a) = \ \varphi_i\ (b) = 0\ (3)$$

It is obvious that the function (1) will satisfy the boundary conditions (2), (3) for any values of constant coefficients.

Substituting (1) into a differential equation, for example into the equation: y”=f(x), we obtain a residual:

$$\varphi_0''\ (x) + \sum_{i=1}^{n} a_i\ \varphi_i''\ (x) - f(x)\ \neq\ 0 \qquad (4)$$

According to the Galerkin method, we require that the discrepancy (4) be orthogonal to the basic ("coordinate") functions $\varphi_i\ (x)$:

$$\int_a^b [\ \varphi_0''\ (x) + \sum_{i=1}^{n} a_i\ \varphi_i''\ (x)]\ \varphi_k\ (x) dx = 0 \qquad (k=1,2,\dots,n)$$

Or

$$\int_a^b \sum_{i=1}^{n} a_i\ \varphi_i''\ (x)\ \varphi_k\ dx = \int_a^b [-\varphi_0''\ (x) + f(x)]\ \varphi_k\ (x) dx$$

Or in expanded form:

$$
\begin{array}{l}
a_1 \int_a^b \varphi_1'' \varphi_1 \, dx + a_2 \int_a^b \varphi_2'' \varphi_1 \, dx + \dots + a_n \int \varphi_n'' \varphi_1 \, dx = \int_a^b (-\varphi_0'' + f) \varphi_1 \, dx \\
.a_1 \int_a^b \varphi_1'' \varphi_2 \, dx + a_2 \int_a^b \varphi_2'' \varphi_2 \, dx + \dots + a_n \int_a^b \varphi_n'' \varphi_2 \, dx = \int_a^b (-\varphi_0'' + f) \varphi_2 \, dx. \\
\dots \\
a_n \int_a^b \varphi_1'' \varphi_n \, dx + a_2 \int_a^b \varphi_2'' \varphi_n \, dx + \dots + a_n \int_a^b \varphi_n'' \varphi_n \, dx = \int_a^b (-\varphi_0'' + f) \varphi_n \, dx
\end{array}
$$

Solving this system of algebraic equations, we also find the solution (1). In general, the functions $\varphi''(x)$ and their second derivatives $\varphi''(x)$ can be non-orthogonal and the Bubnov-Galerkin system and system will not break up into independent separate equations. Moreover, the matrix of coefficients of the system will not be symmetric, i.e. $(\varphi_i'', \varphi_k) \neq (\varphi_k'', \varphi_i)$, где

$$(\varphi_i'', \varphi_k) = \int_a^b \varphi_i'' \varphi_k \, dx$$

scalar products of functions φ_i'' и φ_k

With a sufficiently large (but limited) number of functions (in accordance with the above theorem), a small discrepancy on average is provided. But how close this approximate solution is to the exact one, in general, the question remains open.

Let's continue the consideration of example 2, i.e. solve the equation

$$y'' = \frac{-q}{2EI}(lx - x^2)$$

under boundary conditions y(0)=y(l)=0 by the Bubnov-Galerkin method. We will look for a solution to the problem in the same form:

$$y(x) = \varphi_0(x) + \sum_{i=2}^{n} a_i \sin\frac{i\pi x}{l}$$

Here you can also omit the function , $\varphi_0(x)$, because the sum already satisfies the (zero) boundary conditions of the problem for any a_i function $\varphi_i(x) = \sin\frac{i\pi x}{l}$ and $\varphi_i'' = -(\frac{i\pi}{l})(\frac{i\pi}{l}) \sin\frac{i\pi x}{l}$ coincide with each other up to a multiplier of – , and form an orthogonal system. Therefore, here, too, the Bubnov-Galerkin system decomposes into independent equations:

$$a_i \int_0^l (\frac{i\pi}{l})(\frac{i\pi}{l}) \sin^2\frac{i\pi x}{l} \, dx = -\frac{q}{2EI}\int_0^l (lx - x^2)\sin\frac{i\pi x}{l} \, dx$$

and the formula for determining coincides with the previously obtained Ritz method and the least squares method.

Example. The Galerkin method is used to find an approximate solution of a linear inhomogeneous equation with a variable coefficient

$$y''(x) + x y'(x) + y(x) = 2x \qquad (5)$$

Satisfying the boundary conditions:

y(0)=1, y(1)=0 (6)

We are looking for a solution to the equation in the form:

$$y = (1-x) + a_1(1-x) + a_2 x^2(1-x) + a_3 x^3(1-x) + \dots \qquad (7)$$

$x(1-x) = \varphi_1$; $x^2(1-x) = \varphi_2$; $x^3(1-x) = \varphi_3$ satisfying the boundary conditions (6): y(0)=1, y(1)=0 for any . Substituting (7) into (5), we get the discrepancy:

$$y'': [a_1(-2) + a_2(2-3*2x) + a_3(3*2x-4*3x^2)] \ +$$

$$xy': + [-x + a_1 x(1-2x) + a_2 x(2x-3x^2) + a_3 \ x(3x^2-4x^3)]$$

y: $$+[(1-x) + a_1 \ (x-x^2) + a_2(x^2-x^3) + a_3 \ (x^3-x^4)] =$$

$$(1-2x) + a_1(-2+x-2x^2+x-x^2) + a_2(2-6x+2x^2-3x^3+x^2-x^3) + a_3(6x-12x^2+3x^3-4x^4 +$$

$$+x^3 - x^4) - 2x =$$

$$(1-2x) + a_1(-3x^2+2x-2) + a_2 \ (-4x^3+3x^2-6x+2) + a_3(-5x^4+4x^3-12x^2+6x) - 2x \neq 0 \qquad (8)$$

Multiplying (8) sequentially by $x(1-x) = \varphi_1$; $x^2(1-x) = \varphi_2$; $x^3(1-x) = \varphi_3$

and equating the resulting expressions to zero, we get after integration from 0 to 1:

$$a_1 \int_0^1 (x-x^2)(-3x^2+2x-2)dx + a_2 \int_0^1 (x-x^2)(-4x^3+3x^2-6x+2)dx + a_3 \int_0^1 (x-x^2)(-5x^4+4x^3-12x^2+6x)dx$$

$$a_1 \int_0^1 (3x^4-5x^3+4x^2-2x)dx + a_2 \int_0^1 (4x^5-7x^4+9x^3-8x^2+2x)dx + a_3 \int_0^1 (5x^6-9x^5+16x^4-18x^3+6x)dx = =$$

$$-\int_0^1 (x-x^2)(1-4x)dx = -\int_0^1 (x-5x^2+4x^3)dx = \left(\frac{x^2}{2} - \frac{5x^3}{3} + \frac{4x^4}{4}\right) \Big/_0^1 = \frac{1}{6}$$

or

$$a_1 \left(-\frac{19}{60}\right) + a_2\left(-\frac{3}{20}\right) + a_3\left(-\frac{6}{70}\right) = \frac{1}{6}$$

Where

$$(x-x^2) = \varphi_1$$

Finally we have:

$$\begin{aligned} 133a_1 + 63a_2 + 36a_3 &= -70 \\ 140a_1 + 108a_2 + 79a_3 &= -98 \\ 264a_1 + 252a_2 + 211a_3 &= -210 \end{aligned}$$

Solving a system of algebraic equations, we obtain:

$$a_1=-0.209;\ a_2=-0.7894;\ a_3=0.209$$

Thus, the solution of equation (7) is found:

$$y=(1-x)(1-0.209x-0.7894\chi^2+0.209\chi^3)$$

The Bubnov-Galerkin method has many applications in various fields of science and technology. It is widely used in mathematical modeling, in the theory of elasticity, in hydrodynamics, in the theory of thermal conductivity and many other fields.

One of the advantages of the Bubnov-Galerkin method is its versatility and ease of use. It allows you to solve differential equations of varying complexity and can be adapted to solve specific problems

Thus, the Bubnov-Galerkin method is one of the most effective and widely used methods for solving differential equations. It has many applications in various fields of science and technology and is an important tool for solving complex problems.

Literature

1. Pirnazarov, Gulom Farhodovich. "Symmetric Ram Migrations Style." Procedia of Social Sciences and Humanities 2 (2022): 9-11.
2. Pirnazarov, G. F., & ugli Azimjonov, X. Q. (2022). Determine the Coefficients of the System of Canonical Equations of the Displacement Method and the Free Bounds, Solve the System. Kresna Social Science and Humanities Research, 4, 9-13.
3. Pirnazarov, G. F. (2022). TUTASH BALKA KO’CHISHLAR USULI. BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI, 34-39.
4. Pirnazarov, G. F. (2022). STATIK NOANIQ TIZIMLARNI HISOBLASHDA MATRITSA SHAKLI. BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI, 29-33.
5. Pirnazarov, G. F. (2022). TUTASH BALKALARNI KO’CHISHLAR USULI BILAN QO’ZG’ALMAS YUK TA’SIRIGA HISOBLASH. BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI, 18-22.
6. Pirnazarov, G. F. (2022). RAMALARNI ARALASH VA KOMBINATSIYALASHGAN USULLAR BILAN HISOBLASH. BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI, 23-28.
7. Pirnazarov, G. F. (2022). RAMALARNI ARALASH VA KOMBINATSIYALASHGAN USULLAR BILAN HISOBLASH. BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI, 23-28.
8. Khudjaev, M., Rizaev, A., Pirnazarov, G., & Khojikulov, S. (2022). Modeling the dynamics of a wedge pair under the action of a constant force. Transportation Research Procedia, 63, 458-464.
9. Pirnazarov, G., Khudjaev, M., Khojikulov, S., & Xojakhmatov, S. (2022). Specific methodological aspects of designing railroads protection. Transportation research procedia, 63, 449-457.
10. Фарходович, П. Ғ. (2023, January). Вант Билан Кучайтирилган Шарнирсиз Арка. In " ONLINE-CONFERENCES" PLATFORM (pp. 16-19).

11. Пирназаров, Г. Ф., & угли Озоджонов, Ж. Т. (2022). НО КОНСЕРВАТИВ КУЧЛАР БИЛАН ЮКЛАНГАНДА СТЕРЖЕНЛАРНИНГ БАРҚАРОРЛИГИ ҲАҚИДА. AGROBIOTEXNOLOGIYA VA VETERINARIYA TIBBIYOTI ILMIY JURNALI, 2, 7-12.

12. Pirnazarov, G. N. (2022). Philosophical And Pedagogical Documents of Legal Culture Development at Young People. Eurasian Journal of Humanities and Social Sciences, 9, 32-35.

13. Babakhanova, N. U. (2019). FEATURES OF ACCOUNTING IN RAILWAY TRANSPORT AND ITS PRIORITIES FOR ITS DEVELOPMENT. In WORLD SCIENCE: PROBLEMS AND INNOVATIONS (pp. 33-35).

14. Odilbekovich, S. K., Bekmuratovich, E. A., & Islamovna, M. F. (2023). Requirements for a Railway Operation Specialist on Traffic Safety Issues. Pioneer: Journal of Advanced Research and Scientific Progress, 2(3), 98-101.

15. Халимова, Ш. Р., Мамурова Ф. Я. (2023). Изометрическое и диметрическое представление окружностей и прямоугольников. Miasto Przyszłości , 33 , 128-134.

16. Odilbekovich, S. K. (2023). Optimization of the Ballast Layer on Loaded Freight Cars and High-Speed Lines. Nexus: Journal of Advances Studies of Engineering Science, 2(3), 92-98.

17. Mamurova, F., & Yuldashev, J. (2020). METHODS OF FORMING STUDENTS'INTELLECTUAL CAPACITY. Экономика и социум, (4), 66-68.

18. Islomovna, M. F., Islom, M., & Absolomovich, K. X. (2023). Projections of a Straight Line, the Actual Size of the Segment and the Angles of its Inclination to the Planes of Projections. Miasto Przyszłości, 31, 140-143.

19. Mamurova, F. I. (2022, December). IMPROVING THE PROFESSIONAL COMPETENCE OF FUTURE ENGINEERS AND BUILDERS. In INTERNATIONAL SCIENTIFIC CONFERENCE" INNOVATIVE TRENDS IN SCIENCE, PRACTICE AND EDUCATION" (Vol. 1, No. 4, pp. 97-101).

20. Islomovna, M. F. (2022). Success in Mastering the Subjects of Future Professional Competence. EUROPEAN JOURNAL OF INNOVATION IN NONFORMAL EDUCATION, 2(5), 224-226.

21. Shaumarov, S., Kandakhorov, S., & Mamurova, F. (2022, June). Optimization of the effect of absolute humidity on the thermal properties of non-autoclaved aerated concrete based on industrial waste. In AIP Conference Proceedings (Vol. 2432, No. 1, p. 030086). AIP Publishing LLC.

22. Pirnazarov, G. F., Mamurova, F. I., & Mamurova, D. I. (2022). Calculation of Flat Ram by the Method of Displacement. EUROPEAN JOURNAL OF INNOVATION IN NONFORMAL EDUCATION, 2(4), 35-39.

23. Mamurova, F. I. (2021). The Concept of Education in the Training of Future Engineers. International Journal on Orange Technologies, 3(3), 140-142.

24. Islomovna, M. F. (2023). Methods of Fastening the Elements of the Node. EUROPEAN JOURNAL OF INNOVATION IN NONFORMAL EDUCATION, 3(3), 40-44.

25. Islomovna, M. F. (2023). Engineering Computer Graphics Drawing Up and Reading Plot Drawings. New Scientific Trends and Challenges, 120-122.

26. Khodjayeva, N., & Sodikov, S. (2023). Methods and Advantages of Using Cloud Technologies in Practical Lessons. Pioneer: Journal of Advanced Research and Scientific Progress, 2(3), 77-82.

27. Khusnidinova N. A. Development of Creative Competence of Students in the Process of Teaching Drawing Geometry //New Scientific Trends and Challenges. – 2023. – С. 67-68.